

# Topics in the Implementation and Application of Two-Station Tracking Data Types

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*Two proposed two-station tracking data processing techniques, direct data filtering and differenced data filtering, are analyzed using advanced orbit determination filtering methods. Both techniques are shown to perform comparably, yet direct filtering methods prove to be more sensitive to error model assumptions. Two-station tracking data are shown to be potentially superior to conventional tracking data in determining Deep Space Network tracking station locations.*

## I. Introduction and Summary of Conclusions

The application of two-station tracking techniques to interplanetary navigation has been treated in several preceding *DSN Progress Report* articles (Refs. 1, 2, and 3). The articles present motivating analysis and orbit determination simulation results establishing two-station tracking techniques as an analytically practicable means for improving interplanetary navigation performance when conventional radio techniques are degraded by low declination geometry or poorly modeled spacecraft accelerations.

This article treats two topics: (1) the alternatives available in processing the simultaneous tracking data (the data can either be used directly or be explicitly differenced before it is processed to obtain orbit determination estimates), and (2) the use of two-station tracking to obtain more reliable station location determinations.

The general conclusions obtained from the analysis of these topics can be stated as follows:

- (1) Directly processing simultaneous tracking data and processing explicitly differenced simultaneous tracking data provide a feasible means for obtaining reliable orbit determination estimates.
- (2) The direct processing method, although potentially more powerful than explicitly differencing, places increased demands on the problem of orbit determination filter design. Errors due to spacecraft nongravitational accelerations are virtually eliminated by either technique. However, variations in the design of the filters used in implementing the directly processed data introduce significant variations in the effects due to other orbit determination error sources, e.g., station location errors.

- (3) The use of two-station tracking data eliminates station location determination errors resulting from spacecraft unmodeled acceleration, and therefore two-station techniques promise to be extremely useful in obtaining accurate station location solutions.

## II. Directly Processing Simultaneous Tracking Data

The proper treatment of the simultaneous tracking data is discussed in Ref. 3, which points out that because available geocentric information is deleted from the differenced simultaneous data, better orbit determination performance may result by directly processing the simultaneous data with an orbit determination filter that properly accounts for the noisy spacecraft dynamics (sometimes referred to as process noise). The proper filtering is required to assure that the affected geocentric information is used without permitting the solution to become highly sensitive to acceleration uncertainties. The conventional batch orbit determination filters have only a limited acceleration error modeling capability; thus the proper treatment of the nondifferenced simultaneous data requires advanced filtering techniques, namely, the use of sequential filtering methods.

Although the advanced filtering methods can be expected to be less sensitive to process noise, their design requires an explicit specification of the expected process noise level. This introduces a possible sensitivity of the filter performance to the process noise assumptions used in the filter design. This possibility is investigated in this article by comparing the treatment of simulated tracking data—differenced and nondifferenced simultaneous range and range rate—by a selection of orbit determination filters, differing with respect to the process noise assumptions used in their design.

The results indicate that although the advanced filtering methods prove effective in reducing the sensitivity of the orbit determination errors to process noise, filter performance does vary considerably with respect to the process noise assumptions when the filter is operating on conventional and nondifferenced simultaneous tracking data. The differenced tracking data are virtually unaffected by process noise and, accordingly, the varying filter designs produce only insignificant variations in filter performance.

## III. Filter Nomenclature

The following analysis consists of a comparison of several sequential filters as they are applied to a set of simulated tracking data. Before proceeding with the comparison, it is appropriate to introduce several basic concepts concerning sequential filters. The filtering algorithm used in this analysis is described in detail in Ref. 4. The algorithm is fundamentally equivalent to the Kalman sequential filter (see Ref. 5), although it is implemented in the numerically more stable “square root” form. The Kalman filter is predicated on the following linearized state dynamics and data models:

$$\left. \begin{aligned} x_{k+1} &= \Phi(k+1, k)x_k + \Gamma(k+1, k)u_k \\ z_k &= H_k x_k + v_k, \quad k = 1, \dots, N \end{aligned} \right\} \quad (1)$$

where  $x_k$  is the state vector at a particular stage  $k$ , representing the value of a “to be solved for” parameter vector evaluated at time  $t_k$ . For instance, one may have

$$x_k = \begin{bmatrix} \delta x(t_k) \\ \delta y(t_k) \\ \delta z(t_k) \\ \delta \dot{x}(t_k) \\ \delta \dot{y}(t_k) \\ \delta \dot{z}(t_k) \end{bmatrix}$$

where  $\delta x$ ,  $\delta y$ ,  $\delta z$  and  $\delta \dot{x}$ ,  $\delta \dot{y}$ ,  $\delta \dot{z}$  are displacements in the components of spacecraft position and velocity relative to a nominal position and velocity, and

$$\Phi(t_{k+1}, t_k) = \frac{\partial(x(t_{k+1}), \dots, \dot{z}(t_{k+1}))}{\partial(x(t_k), \dots, \dot{z}(t_k))}$$

is the state transition matrix.

The vector  $x_k$  may also include other solve for parameters such as station locations and spacecraft accelerations. The  $u_k$  is the process noise vector, a sequentially uncorrelated stochastic variable with covariance function

$$Eu_k u_j^T = Q_k \delta_{kj} \quad (2)$$

Process noise allows the introduction of statistical uncertainty into the state dynamics, and as in the following application, can represent the effects of random accelerations on a spacecraft.

The data equation in Eq. (1) represents the navigation data in linearized form, i.e.,

$z_k = (\text{observed data} - \text{computed data}), \text{ available at the } k\text{th stage, and}$

$$H_k = \frac{\partial(\text{observable})}{\partial x_k}$$

The  $z_k$  is a vector representing data taken over times  $t, t_k < t < t_{k+1}$ . The  $v_k$  is assumed to be a sequentially uncorrelated data noise vector with covariance function

$$E v_k v_j^T = R_k \delta_{kj} \quad (3)$$

The statistical assumptions concerning Eq. (1) are fully determined upon the specification of an a priori covariance for the state vector; i.e.,

$$E x_1 x_1^T = P_1 \quad (4)$$

Given a model (Eq. 1) the specification of Eqs. (2), (3), and (4) determines a minimum variance sequential filter for estimating  $x_k$  given the data  $z_j, j = 1, \dots, k$ . A sequential formula (Ref. 5) for the estimate  $\hat{x}_k$  is given by

$$\hat{x}_{k+1} = \Phi(t_{k+1}, t_k) (\hat{x}_k + K_k [z_k - H_k \hat{x}_k]), k = 1, \dots, N \quad (5)$$

where the filter "gain"  $K_k$  is given by

$$K_k = P_k H_k^T (H_k P_k H_k^T + R_k)^{-1} \quad (6)$$

with the filter covariance of estimate errors  $P_k$  determined by the following sequential relationship:

$$P_{k+1} = \Phi(t_{k+1}, t_k) (I - K_k H_k) P_k (I - K_k H_k)^T \Phi(t_{k+1}, t_k)^T + \Gamma(t_{k+1}, t_k) Q_k \Gamma(t_{k+1}, t_k)^T, \quad k = 1, \dots, N \quad (7)$$

These formulas are determined initially by assumed values for  $P_1$  and  $\hat{x}_1$ .

Under the assumption that  $Q_k = 0$  for all  $k$ , the model is easily reformatted into the familiar batch or minimum variance parameter estimation form

$$z = Ax + \epsilon$$

where

$$z = \begin{pmatrix} z_1 \\ \vdots \\ \vdots \\ \vdots \\ z_n \end{pmatrix}$$

$$\epsilon = \begin{pmatrix} v_1 \\ \vdots \\ \vdots \\ \vdots \\ v_n \end{pmatrix}$$

and  $x$  may be any particular  $x_k, k = 1, \dots, N$  since any one state vector value now uniquely determines all other values.

## IV. Filter Performance and Q Magnitude

It is pointed out in Section III that the conventionally implemented batch filter can be considered as a sequential filter with  $Q_k = 0, k = 1, \dots, N$ . The restrictive error modeling capability of the batch filter may allow degradations in filter performance in certain applications. Specifically, in the case of a spacecraft affected by random accelerations, the conventional batch navigation filters have exhibited significant performance degradations (Ref. 6.)

With the application of sequential filters, with  $Q_k \neq 0$ , a new problem arises: the selection of the proper sequence of covariance matrices  $Q_k, k = 1, \dots, N$ . This problem is accompanied by the problem of selecting the proper sequence of data weights  $R_k, k = 1, \dots, N$ , a problem that can be handled somewhat arbitrarily for the batch filter. Often the general character of the  $R$  and  $Q$  matrices is reasonably well known with only their "magnitude" being uncertain. That is, assume matrices  $\bar{R}_k$  and  $\bar{Q}_k$  can be specified so that "proper" specification of  $R_k$  and  $Q_k$  entails only the selection of scalars  $r$  and  $q$  so that

$$R_k = r \bar{R}_k \text{ and } Q_k = q \bar{Q}_k, k = 1, \dots, N$$

A simple yet important fact concerning the relation between the values of  $r$  and  $q$  and filter performance can be stated as follows: Filter implementation as specified by the gain sequence  $K_k, k = 1, \dots, N$ , is unaffected by the absolute magnitudes of  $r$  and  $q$ , and depends only on their ratio  $q/r$ . Thus, given  $\bar{R}$  and  $\bar{Q}$ , the variations in filter performance (with respect to a fixed and not necessarily known error environment) can be ascertained by investigations of the effects of  $q$ (or  $r$ ) variations alone. This result can be demonstrated inductively by noting that if

$$P_1 = q F_1 \left( \frac{q}{r} \right) + r G_1 \left( \frac{q}{r} \right)$$

for particular functions  $F_1(\bullet)$  and  $G_1(\bullet)$ , then

$$K_1 = K_1 \left( \frac{q}{r} \right)$$

by Eq. (6), and if

$$P_k = qF_k \left( \frac{q}{r} \right) + rG_k \left( \frac{q}{r} \right)$$

then

$$K_k = K_k \left( \frac{q}{r} \right)$$

$$P_{k+1} = qF_{k+1} \left( \frac{q}{r} \right) + rG_{k+1} \left( \frac{q}{r} \right)$$

by Eq. (7).

## V. Analysis of Simulated Tracking Data

The effect of varying filter  $Q$  levels is analyzed in the following through the evaluation of sequential filters in terms of their operation on simulated tracking data. The simulated data were generated from rigorous orbit determination analysis software and are contained in two sets. The first contains simulated data residuals that result from a spacecraft being perturbed by random accelerations with a standard deviation of  $10^{-12}$  km/s<sup>2</sup> and an autocorrelation time of 5 days. The data are the same set used in Ref. 3: simultaneous and conventional tracking strategies applied to the encounter minus 30 days to encounter, data arc of Viking '75 Mission B.

Data set II is data set I with data residuals due to station location errors superimposed. The data-taking strategies and spacecraft trajectory used in generating the simulated data are detailed in Ref. 3.

The station location errors used to generate data set II are presented in Table 1. The errors are arbitrarily selected and are intended to be representative of the station location errors present for the Viking mission in 1975-1976. Note that the errors contain a bias and a random component, the random component chosen to represent apparent station location error effects such as timing, polar motion, and residual data transmission media calibration effects. The simulated differenced and nondifferenced range rate and range residuals for data sets I and II are shown in Figs. 1-4.

Four filters are applied to each of the data sets. Each filter solved for only the position and velocity of the spacecraft. The filters differ by their assumed  $Q$  levels. The  $Q$  matrices are determined by the assumption that the spacecraft experiences three-axis, spherically distributed piecewise constant accelerations, constant over 1 day and uncorrelated from day to day. The " $Q$  level" is specified by the assumed single-axis acceleration standard deviations, with the four  $Q$  levels being 0,  $10^{-12}$ ,  $10^{-11}$  and  $10^{-10}$  km/s<sup>2</sup>. The zero- $Q$  level filter processes the tracking data as if it were a batch filter.

The results of the simulated data analysis are presented in Figs. 5-7. The results are presented in terms of estimates and standard deviations of the  $B$ -plane parameter  $B \cdot R$ , a position component particularly difficult to determine for the Viking mission B. The errors are presented for varying data arcs, from epoch to encounter minus 0.5, 5, 10, 15 and 20 days (encounter occurs at 30 days past epoch.) Since the simulated tracking data are due solely to modeled tracking data errors, the estimates ( $\Delta B \cdot R$ ) are resulting navigation errors. The  $\Delta B \cdot R$  (Deep Space Station (DSS)) are the estimate errors due to station location errors alone (data set II minus data set I) while the  $\Delta B \cdot R$  (attitude control acceleration (ATA)) are the estimate errors due to acceleration errors alone (data set I.)

Figure 5 shows that, when using conventional data, the effects of the acceleration errors on the  $B \cdot R$  estimates are significantly reduced with each increase in the  $Q$  level. Note, however, that the  $Q$ -level variations produce large differences in the responses to the station location errors,  $\Delta B \cdot R$  (DSS). This is a disquieting characteristic in that the effects due to station location errors are usually expected to be perhaps large but stable with respect to various filter implementations.

Assessment of the general nature of these effects cannot be obtained through analysis of a few simulated data sets, as presented here; general results will require a detailed filter covariance analysis. The implication of these results are clear, however: that sequential filters when acting on conventional tracking data can reduce the effects of random spacecraft accelerations, at the expense of variations in the sensitivity to station location errors.

Figure 6 presents the filter performance results for simultaneous range and range rate data. Reference 3 points out that although the simultaneous data improve

station location error sensitivities (for near-zero declination spacecraft), the tight range specification greatly magnifies the random acceleration effects. This effect is well illustrated in Fig. 6. Note that the simultaneous data filter sigmas are more sensitive to the  $Q$  level assumptions than the conventional data filter sigmas. The batch filter is seen to be very sensitive to random acceleration, yet sufficiently large  $Q$  levels essentially eliminate the acceleration effects. The station location error effects, unfortunately, show even larger variations with changing  $Q$  level.

Figure 7 shows the differenced range and range rate  $B \cdot R$  sigmas and errors for the batch filter only. The  $Q \neq 0$  filters are not shown because their estimates are virtually unaffected by  $Q$  level. Similarly, the error responses due to random accelerations are not shown since they are all essentially zero.

One can conclude from these results that although the simultaneous range and range data may be, in principle, potentially superior to differenced data, initial applications using advanced filtering methods indicate that the results are particularly sensitive to filter model assumptions and not clearly superior to processing the differenced data with conventional filters.

## VI. Station Location Determinations From Two-Station Tracking

In the analytical analysis of a single pass of radio tracking data performed in Ref. 1, it was shown how the effects of unmodeled spacecraft (S/C) accelerations can be expressed in terms of equivalent station location errors. Thus it should be expected that unmodeled S/C accelerations will degrade station location solutions obtained from radio metric tracking data. Since, as shown above, the effects of unmodeled S/C accelerations can be essentially eliminated by differencing simultaneous two-station data, such techniques may be useful in improving station location solutions.

The use of two-station simultaneous tracking data, in particular very long baseline interferometer (VLBI) measurements, is by no means a novel approach in determining station locations. The general effectiveness of such techniques is well established (see Ref. 7), and yet the fact that these methods offer special advantages in almost eliminating the station location solution sensitivities to S/C accelerations is not widely realized. The S/C

acceleration errors may include gravitational as well as nongravitational effects. For example, seemingly small S/C position errors or gravitational model errors can give rise to significant S/C acceleration discrepancies during planetary encounter and satellite phases of a mission.

To obtain some idea of the station location sensitivities to the unmodeled S/C accelerations, a brief "consider" covariance analysis was performed for the Saturn portion of a Jupiter-Saturn mission described in Ref. 8. This analysis was based upon a batch filter which estimated or considered the parameters shown in Table 2. For comparison, parallel analyses were carried out for long arc data sets which, as shown in Table 3, were identical except that in case 1 the simultaneous data are used directly and in case 2 the simultaneous data are explicitly differenced before processing.

The effects of the random data noise on the station locations solutions for cases 1 and 2 are expressed in terms of the formal standard deviations given in Fig. 8. The formal standard deviations associated with the differenced data are typically four times larger than the same quantities associated with the nondifferenced data. It is expected that the differenced data formal statistics will be larger, because, as mentioned previously, the differencing procedure deletes the acceleration information.

Figure 8 also contains the sensitivities of the station location solutions to errors in the planetary gravitational constant (GM) and to a constant S/C acceleration in the Sun-to-S/C direction. These sensitivities are represented in terms of errors in the station location solutions produced from errors in GM and S/C accelerations of magnitude  $0.6 \times 10^2 \text{ km}^3/\text{s}^2$  and  $0.5 \times 10^{-12} \text{ km/s}^2$ , respectively. These sensitivity bars represent the effect of the particular error source only, and scale directly with the magnitude of the error. An examination of Fig. 8 clearly shows that if the unmodeled accelerations are present at the indicated levels, the long arc station location solutions will contain catastrophic errors unless the data are differenced.

Undoubtedly, it is possible to process conventional data so that the station location sensitivities to unmodeled S/C accelerations can be reduced from those shown in Fig. 8. However, Fig. 8 does show that the use of differenced data may be extremely useful in obtaining accurate station location solutions and that the concept certainly deserves a more detailed investigation.

## References

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**Table 1. Assumed station location errors**

Error	Bias value, $m$			Random component standard deviation <sup>a</sup>
	California	Australia	Spain	
Spin axis	1.5	-1.5	1.5	1.0
Radius				
Longitude	3.0	3.0	3.0	1.0
Z height	10.0	10.0	10.0	1.0

<sup>a</sup>The random components are assumed to be due to gaussian first-order markov processes with autocorrelation times equal to 3 days.

**Table 2. Estimated and considered parameters for station location solutions**

Estimated parameters	A priori	"Consider" parameters	"Consider" $\sigma$
S/C position	$10^7$	Constant acceleration in Sun-S/C direction	$0.5 \times 10^{-12} \text{ km/s}^2$
S/C velocity	1 km/s		
Planetary ephemeris	1000 km		
DSS 14, 42, 61 locations		Planetary GM	$0.6 \times 10^2 \text{ km}^3/\text{s}^2$ ( $2 \times 10^{-3}\%$ )
Spin axis $r_s$	1 m		
Longitude $\lambda$	2 m		

**Table 3. Case numbers and data sets**

Case	Description	Data <sup>a</sup> ( $E - 120 \rightarrow E + 30$ )	Data weight <sup>b</sup>	Stations
1	3 stations	$\rho$	3 m	14, 42, 61
		$\dot{\rho}$	1 mm/s	14, 42, 61
2	Differenced	$\rho$	300 m	42
		$\dot{\rho}$	100 mm/s	42
		Diff $\rho$	3 m	14, 42, 61
		Diff $\dot{\rho}$	1 mm/s	14, 42, 61

<sup>a</sup>For exact tracking pattern, see Ref. 8.  $\rho$  = range,  $\dot{\rho}$  = range-rate, diff = explicitly differenced data.

<sup>b</sup>See Ref. 8.

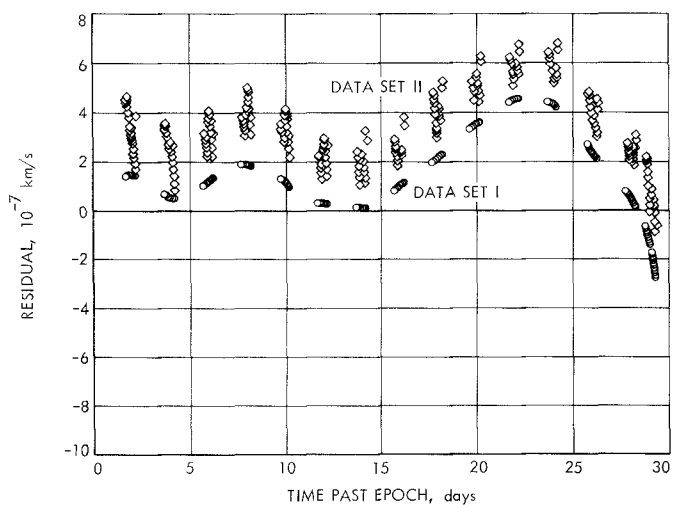


Fig. 1. Range rate residuals

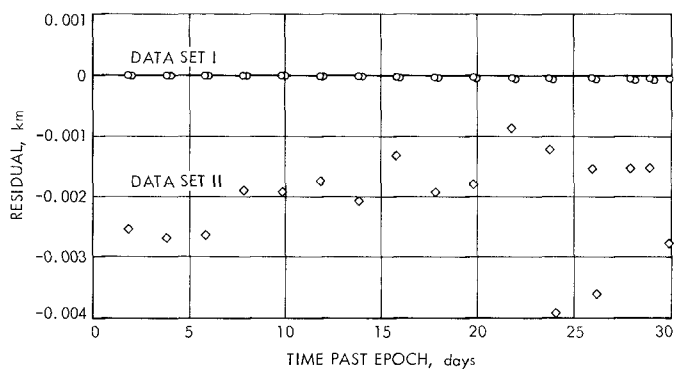


Fig. 4. Differenced range residuals

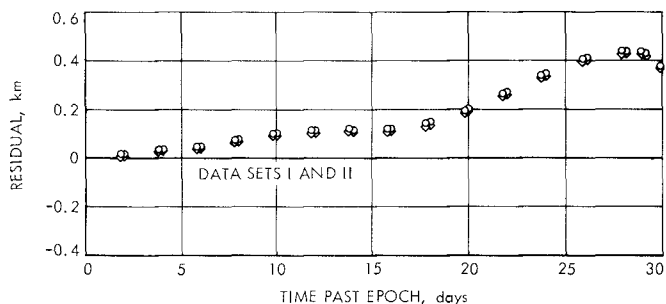


Fig. 2. Range residuals

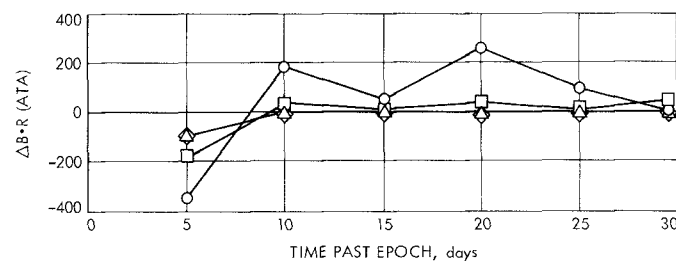
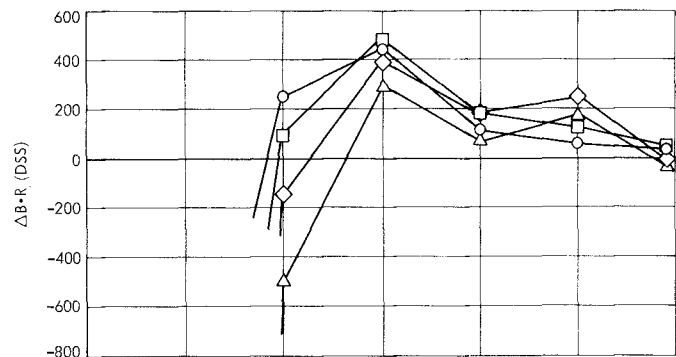
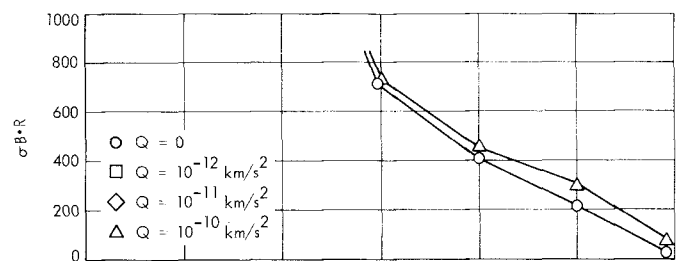


Fig. 5. Conventional range and range rate

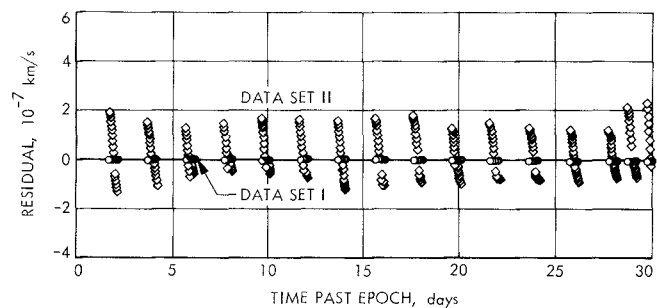


Fig. 3. Differenced range rate residuals



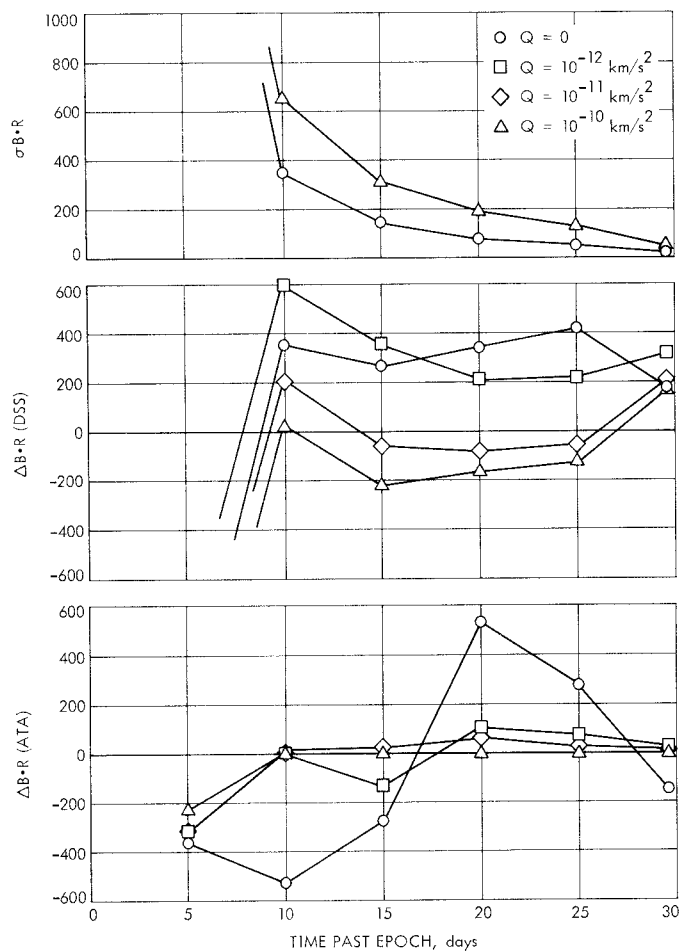


Fig. 6. Simultaneous range and range rate

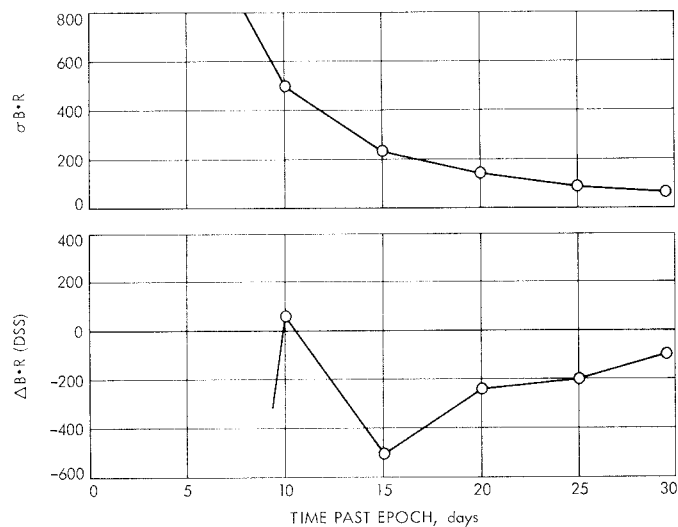


Fig. 7. Differenced range and range rate

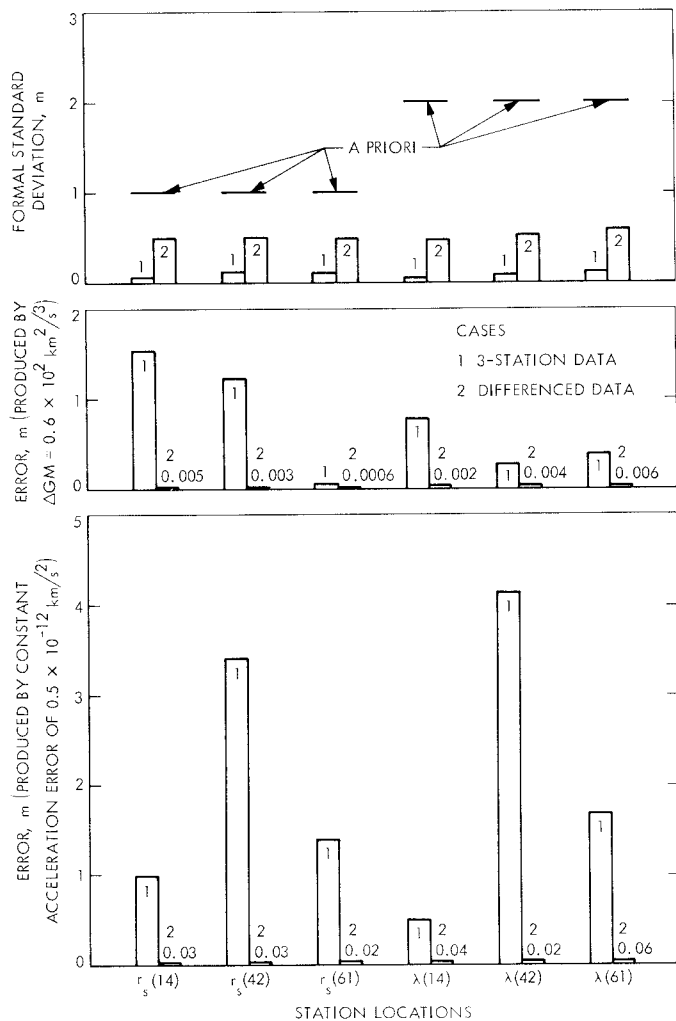


Fig. 8. Station location formal standard deviations and errors produced by GM and constant acceleration errors